THERMAL CONDUCTIVITY OF FIBROUS SYSTEMS

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The process of heat transfer in fibrous materials with a random structure is examined and a method is proposed for calculating the effective thermal conductivity as a function of the characteristic parameters of the system (thermal conductivities of the components, their volume concentration, temperature, pressure of the interstitial gas, etc.).

A distinctive feature of fibrous systems is the low volume concentration of the solid component (from 20 to 0.1%) and its considerable elongation (ratio of fiber length to diameter more than 100).

Fibrous materials can be divided into three classes according to their structure: 1) systems with a random fiber distribution (wool, felt); 2) systems with an ordered fiber distribution (cloths, mats); 3) combined systems, combinations of random and ordered fiber distribution (alternating layers of wool and cloth, napped materials, etc.). We will examine an analytic method of determining the effective thermal conductivity of a fibrous system with a random fiber arrangement.

Fibrous system model. A realistic model is one with a random arrangement of infinitely long cylinders. We assume that the effective thermal conductivity of the actual fibrous system (statistical fiber mixture) and the effective thermal conductivity of an ordered structure are the same if the coefficients of thermal conductivity and the concentrations of the components of the actual and ordered structures are the same. The model of an ordered structure is shown in Fig. 1a.

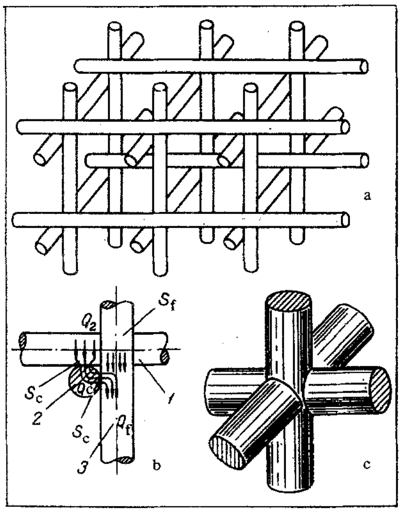

Fig. 1. Model of fibrous system: a) general view of model with cylindrical fibers; b) fiber contact; c) ordered system.

Consider first heat transfer in this model at the fiber contact points 1–2 and 2–3. The contact area S_c (Fig. 1b) between fibers is small compared with the cross section S_f of the fibers: $S_c \leq 1 \cdot 10^{-4} S_f$ [1]. Accordingly, the total heat flux Q_c through contacts

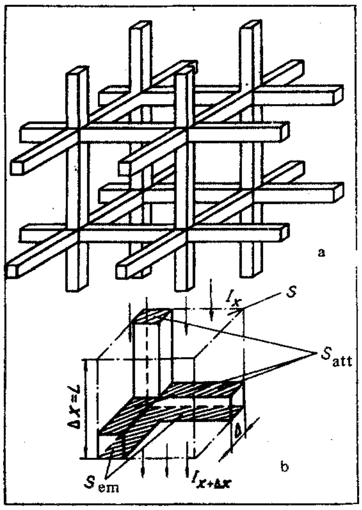

Fig. 2. Model of fibrous system in the form of two interpenetrating components with square bars of constant cross section: a) general view of system; b) unit cell.

1–2 and 2–3 is negligibly small as compared with the flux Q_f along the fibers 3 and through the interstitial gas Q_g. This permits to simplify the model to a system of intersecting cylinders forming a cubic space lattice. Part of this system is shown in Fig. 1c.

Notice that replacing the cylinders with square fibers of the same cross section will not affect much the effective thermal conductivity of the system (Fig. 2a), although the heat-transfer analysis will be considerably simplified. The model of interpenetrating components with square bars of constant cross section shown in Fig. 2a was investigated in [2].

The thermal conductivity λ of the system is related with the thermal conductivity λ_1 of the fibers, the thermal conductivity λ_2 of the interstitial gas, and its volume concentration m_2 as follows [2]

$$\lambda = \lambda_1 \left[c^2 + \nu (1-c)^2 + \frac{2\nu c (1-c)}{\nu c + (1-c)} \right], \quad \nu = \frac{\lambda_2}{\lambda_1}, \qquad (1)$$

where the parameter c depends on the volume concentration as

$$m_2 = 2c^3 - 3c^2 + 1, \qquad (2)$$

whose solution is

$$c = 0.5 + A\cos\frac{\varphi}{3}, \quad (3)$$

with

$$0 \leqslant m_2 \leqslant 0.5,\ A = -1,\ \varphi = \arccos(1-2m_2),$$
$$0.5 \leqslant m_2 \leqslant 1.0,\ A = 1,\ \varphi = \arccos(2m_2-1),$$
$$270^\circ \leqslant \varphi \leqslant 360^\circ.$$

Part of the unit cell of the system with long-range order shown in Fig. 2a is represented in Fig. 2b. It can be shown that $c = \Delta/L$ where Δ is the thickness of the bar, and L the height of the unit cell.

The thermal conductivity λ_2 of the gas is determined from the condition that the heat conduction σ_2 of the gas component in the unit cell should equal the sum of molecular σ_{2m}, convective σ_{2c}, and radiative σ_{2r} heat transfer

$$\sigma_2 = \sigma_{2m} + \sigma_{2c} + \sigma_{2r}. \quad (4)$$

If L is the dimension of the unit cell in the direction of heat flow and S is the cross-sectional area of the cell, then

$$\sigma_2 = \frac{\lambda_2 S}{L} = \frac{\lambda_{2m} S}{L} + \frac{\lambda_{2c} S}{L} + \frac{\lambda_{2r} S}{L},$$

whence

$$\lambda_2 = \lambda_{2m} + \lambda_{2c} + \lambda_{2r}, \quad (5)$$

where λ_{2m}, λ_{2c}, λ_{2r} are the molecular, convective, and radiative components of the thermal conductivity coefficient.

We will examine the individual components of the thermal conductivity of the gas in greater detail.

Under natural conditions there is almost no convection in the system and accordingly we can set $\lambda_{2c} = 0$ [3]. The molecular component λ_{2m} can differ from the thermal conductivity λ_0 at normal pressure H_0 [4]

$$\lambda_{2m} = \frac{\lambda_0}{1 + \dfrac{B}{H\delta'}},$$

$$B = \frac{4\dfrac{c_p}{c_v}}{1 + \dfrac{c_p}{c_v}} \frac{1}{\Pr} \frac{2-a}{a} H_0 \frac{\Lambda_\infty}{1 + \dfrac{C}{T}}, \quad (6)$$

where c_p/c_v is the ratio of specific heats; a is an accommodation factor; Pr is the Prandtl number under normal conditions of pressure and temperature T; H is the interstitial gas pressure; Λ_∞ is the molecular mean free path at high temperature; and C is the Sutherland constant.

We define the pore dimension δ' as the distance between two solid surfaces traversed by a gas molecule. The minimum value of this distance is $\delta_1' = 2(L - \Delta)$; the maximum value is the total thickness of the fibrous system l, i.e.,

$$\delta_2' = l.$$

Consequently,

$$\delta_1' \leqslant \delta' \leqslant \delta_2'.$$

For small fiber volume concentrations the value of δ' approaches l.

For a quantitative estimate of the effective pore size we will consider a somewhat modified model, in which each successive fiber in the direction of heat flow is displaced perpendicular to the heat flow by an amount equal to its diameter. This is equivalent to the displacement of each successive unit cell by the width of a bar. We believe that this model is closer to the actual structure. For such a model δ' can be found from

$$\delta' = 2L\frac{2L - 2\Delta}{2\Delta} - 2\Delta = 2\Delta\left[\frac{1-c}{c^2} - 1\right]. \quad (7)$$

Equation (7) is a rough approximation.

Since the cylindrical and square cross sections are equal, it follows that Δ is related to the fiber diameter D by

$$\Delta = \frac{\sqrt{\pi}}{4} D. \quad (8)$$

Using (8), we write (7) as

$$\delta' = \frac{\sqrt{\pi}}{2}\left[\frac{1-c}{c^2} - 1\right] D \simeq 0.89D\left[\frac{1-c}{c^2} - 1\right]. \quad (9)$$

For very thin fibers, even at normal external gas pressure, the mean free path may exceed δ'. This should be kept in mind in determining λ_{2m}. Relations (6) and (9) allow this effect to be taken into account. In most cases the mean free path is much less than the distance between fibers, i.e.,

$$\lambda_{2m} = \lambda_0.$$

Radiative component. Radiative heat exchange constitutes a considerable fraction of the total transfer in fibrous systems owing to the low fiber concentration. Moreover, the radiative heat exchange is affected by the emissivity of the surfaces bounding the fibrous system and the distance between them [5–9]. Accordingly, in deriving the functional relation linking the thermal conductivity of fibrous systems with the characteristic parameters, we will consider heat transfer not in the unit cell but across the system as a whole with allowance for the characteristics of the bounding surfaces. Consider a homogeneous isotropic medium with the following integral characteristics: absorption coefficient α, scattering coefficient σ, and some thermal conductivity coefficient λ_c. The latter takes into account molecular heat transfer through the gas and conductive transfer along the fibers. Assume that there is no convective heat transfer, and that the surfaces of the isotropic medium are parallel and isothermal.

This problem was investigated by a number of authors, in particular, for a random fibrous system [8]. However, with this result it is not possible to evaluate the radiative component of the effective thermal conductivity of the system without resorting to an experimental determination of certain parameters, in particular, the attenuation factor.

Using the general formulation of the problem and the individual results reported in [6, 7], we will examine an analytic method of determining the attenuation factor in the case of total absorption of the radiation.

In the general case, the attenuation of radiation intensity in a medium is determined by absorption and scattering. The attenuation factor $\beta(\nu)$ is equal to the sum of the absorption $\alpha(\nu)$ and scattering $\sigma(\nu)$ coefficients

$$\beta(\nu) = \alpha(\nu) + \sigma(\nu). \tag{10}$$

In the case of total absorption ($\sigma = 0$) the attenuation factor is equal to the absorption coefficient:

$$\beta(\nu) = \alpha(\nu).$$

The mechanism of heat transfer in a homogeneous isotropic medium with zero scattering was examined in detail in [9], where an expression was given for the radiative component of the thermal conductivity as a function of absorption coefficient α, the temperature of the medium T, the thickness of the fibrous system l, and the emissivity of the surfaces bounding the system ε:

$$\lambda_r = \frac{16}{3}\,\frac{\sigma_S T^3 Y}{\alpha},$$

$$Y = 1 - \frac{3}{4\tau}\,[1 - 4K_3(\tau)] - \frac{2}{3\tau}\times$$

$$\times(1-\varepsilon)\,\frac{[1-3K_4(\tau)]^2}{1+2(1-\varepsilon)K_3(\tau)}, \tag{11}$$

where σ_S is the Stefan-Boltzmann constant $\sigma_S = 5.67 \cdot 10^{-8}\ \text{W/m}^2 \cdot \text{deg}$;

$$K_n(\tau) = \int_0^1 \exp\left(-\frac{\tau}{\mu}\right)\times$$

$$\times\,\mu^{n-2}\,d\mu,\ \mu = \cos\theta,\ \tau = \alpha l.$$

Values of $Y = Y(\varepsilon, \tau)$ were tabulated in [9]; a graph of $\tau = \tau(\varepsilon, Y)$ is presented in Fig. 3.

We will find the relation between the absorption coefficient and the geometric parameters of the fibrous system, which we represent in the form of two interpenetrating components. By definition [10]

$$\beta = \frac{S_{\text{att}}}{S\,\Delta x}, \tag{12}$$

where S_{att} is the effective cross section of the particles attenuating the radiation on an area S in a layer of thickness Δx equal to the mean distance between particles (Fig. 2b).

For the unit cell in question (Fig. 2b) the cross section parameter in a plane perpendicular to the heat flow is $S = L^2$, while Δx corresponds to the edge of the unit cell ($\Delta x = L$), and the attenuation surface

$$S_{\text{att}} = L^2 - (L-\Delta)^2. \tag{13}$$

With (8), (4), and (13), (11) is reduced to

$$\beta = \alpha = \frac{L^2-(L-\Delta)^2}{L^2 L} = \frac{c^2(2-c)}{\Delta} = 2.26\,\frac{c^2(2-c)}{D}, \tag{14}$$

where $c = \Delta/L$ is the parameter associated with the gas volume concentration m_2 in Eq. (3).

On the basis of (10) and (14)

$$\lambda_r = 0.134\,\frac{D}{c^2(2-c)}\left(\frac{T}{100}\right)^3 Y. \tag{15}$$

If the attenuation depends exclusively on the scattering ($\alpha = 0$), then [6]

$$\lambda_r = 0.226\,\frac{l}{\frac{2}{\varepsilon} - 1 + \sigma l}\left(\frac{T}{100}\right)^3. \tag{16}$$

In most cases the scattering coefficient is determined experimentally. However, if the refractive index of the fiber material is known, σ can be estimated from the formula [6]

$$\sigma(\nu) = \frac{4(1-m_2)}{\pi D}\,k_{sc}, \tag{17}$$

where k_{sc} is the scattering coefficient for an individual spherical particle with refractive index $n(\nu)$ equal to the refractive index of the fiber and with diameter equal to the fiber diameter. The scattering coefficients for spherical particles were calculated in [11] as a function of the complex refractive index and the relation between the particle diameter and the wavelength of the incident radiation.

In particular, if the fiber diameter is much less than the wavelength of the incident radiation, k_{sc} for the given wavelength may be calculated from [12, 13]

$$k_{sc} = \frac{8}{3}(\pi D\nu)^4\left|\frac{n^2-1}{n^2+1}\right|. \tag{18}$$

Here, n is the complex refractive index of the material.

If the fiber diameter is much greater than the wavelength of the incident radiation, then

$$k_{sc} = 2\left(1 + \frac{0.4}{\pi D\nu}\right). \tag{19}$$

In the event that the attenuation is caused by both absorption and scattering, then, if

$$T_1 \gg \frac{3(T_1-T_2)}{\alpha l}, \tag{20}$$

where T_1 and T_2 are the temperatures of the bounding surfaces, the following expression can be used for the radiative component [6]:

$$\lambda_r = \frac{4\sigma_S T_\varepsilon^3}{(\beta+\sigma)}\times\left\{\frac{(\beta+\sigma)\varepsilon + \sqrt{\beta^2-\sigma^2}\,(2-\varepsilon)}{2\alpha(1-\varepsilon) + \beta\varepsilon^2 + \sqrt{\beta^2-\sigma^2}\,\varepsilon(2-\varepsilon)}\right\}. \tag{21}$$

We have proposed a number of formulas for calculating the radiative components of the thermal-conductivity coefficient as a function of the properties of

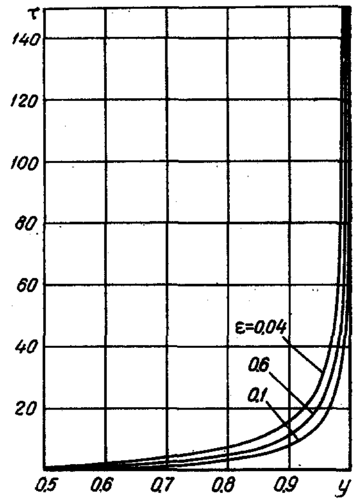

Fig. 3. Coefficient Y as a function of the optical path τ for various emissivities ε of the bounding surfaces.

the fibrous system. However, for most fibrous materials it is possible to calculate the radiative component of thermal conductivity from Eq. (15) taking only absorption into account.

The effective thermal-conductivity coefficient of an orderless fibrous system can be calculated from Eqs. (1), (5), (6), (9), and (11). A comparison with the experimental data shows satisfactory agreement.

NOTATION

λ is the effective thermal conductivity of fibrous systems, λ_1 is the thermal conductivity of the fibers; λ_2 is the thermal conductivity of the interstitial gas; m_2 is the fiber volume concentration; D is the fiber diameter; δ' is the distance between fibers; β, α, σ are the attenuation factor, absorption, and scattering coefficients; ε is the emissivity of the reflecting surfaces; l is the thickness of the fibrous sytem; and T is the temperature of the fibrous system.

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